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for $h \leq h_1$. In this case

$$p \geq P \geq 0, \quad P = p\pi = 0.$$

4. We can summarize the results here obtained by saying that the theorem of compound probabilities holds true when p and π are increasing functions of their arguments, or increase in certain intervals and are constant elsewhere. But this condition is always realized, as we cannot have

$$p(a, h_1) = p(a, h_2), \quad \pi(\alpha, k_1) = \pi(\alpha, k_2)$$

unless the functions p and π are constant in the intervals (h_1, h_2) and (k_1, k_2) respectively.

BITS OF HISTORY ABOUT TWO COMMON MATHEMATICAL TERMS.

By G. A. MILLER, University of Illinois.

In 1841 A. L. Cauchy defined the term *indicator* (indicateur) corresponding to a given modulus n as the exponent to which a positive integer m relatively prime to n belongs mod n , and gave tables for the determination of the maximum indicator I corresponding to a given modulus n , where n may be replaced successively by a series of positive integers. For instance,

$n =$	2	3	4	5	6	7	8	9	10	11	12	13
$I =$	1	2	2	4	2	6	2	6	4	10	2	12

About four years later E. Prouhet published a note in volume 5 of the *Nouvelles Annales de Mathématiques*, page 75, in which he defined the term indicator of n as the number $\phi(n)$ of the positive integers less than n and prime to n . He added incorrectly that this term had not been employed previously in mathematics. It seems somewhat singular that this later definition came into common use, especially since Cauchy was a more noted mathematician than Prouhet and this definition was due to ignorance on the part of its author.

In the language of group theory Cauchy's definition of indicator is equivalent to that of the order of the totitives of n , while that of Prouhet is equivalent to that of the order of the group formed by these totitives mod n . The I of n is therefore always a divisor of $\phi(n)$ and a necessary and sufficient condition that I of n is equal to $\phi(n)$ is that n has a primitive root, or that the group of totitives of n is cyclic. In general, the value of I is equivalent to the order of the largest cyclic subgroup contained in this group.

As an instance where a later definition of a common term had an entirely different fate we may refer to the definition of *simple group* found on page 65, volume 20, *Proceedings of the London Mathematical Society*. According to this definition a group is called *simple* when it is both cyclic and its order is a power of a prime number, while the common definition of simple group as a group which

does not involve any invariant subgroup besides the identity had been used by C. Jordan about twenty years earlier.

No one else seems to have adopted this later definition of simple group. Its implicit use in L. E. Dickson's *History of the Theory of Numbers*, volume 1, page 131, is doubtless due to an oversight. Curiously H. W. L. Tanner continued the use of his unfortunate definition in an article published about seven years later in volume 27 of the same journal, page 331.

The mathematical literature contains numerous definitions due to a lack of knowledge on the part of their authors. The different fates of the two cited above may serve to illustrate how difficult it is to deal with some of these definitions. The fact that they are potential sources of error, especially when they appear in periodicals which are frequently used, makes it undesirable to ignore even those which are not adopted by others.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

REPLIES.

36. A number of Discussions have been published in this department relating to cubic and biquadratic equations (cf. Vol. XXXV, p. 29, pp. 268-269 and 343-347; Vol. XXIV, pp. 136-137 and 436-439; Vol. XXIII, pp. 314-315). Below are given a number of questions sent in by Professor Harris Hancock of the University of Cincinnati which relate to the cubic and biquadratic and might, perhaps, more properly be proposed as problems were it not for the advantage to be gained, if possible, by treating them all, or at least several of them, in one discussion.

1. For what values of n can $\cos 2\pi/n$ be expressed in the form $(a + \sqrt{b})/c$ where a, b and c are integers?

2. Write the biquadratic in the form $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$. Show that its reducing cubic may be expressed by means of a determinant of the third order which when expanded is

$$4y^3 - g_2y - g_3 = 0,$$

where $g_2 = ae - 4bd + 3c^2$, and

$$g_3 = \begin{vmatrix} a, b, c \\ b, c, d \\ c, d, e \end{vmatrix} = ace - ad^2 - eb^2 - c^3 + 2bcd.$$

3. For the same biquadratic show that

$$a^3(x_0 + x_1 - x_2 - x_3)(x_0 + x_2 - x_1 - x_3)(x_0 + x_3 - x_1 - x_2) = 32(3abc - a^2d - 2b^3)$$

without making any use of symmetric functions, where x_0, \dots, x_3 are the roots of the biquadratic equation.

4. If x_0, x_1, x_2 are the roots of a cubic and D its discriminant, show that x_1 is a rational function of x_0 and \sqrt{D} . Derive a much simpler relation than that given in Serret, *Cours d'Algèbre Supérieure*, 5th ed., No. 511.

5. If x_0, x_1, x_2, x_3 are the roots of a biquadratic, D its discriminant, e_1, e_2, e_3 the roots of the reducing cubic, show that x_1 is a rational function of x_0, e_1, e_2, e_3 and consequently also of x_0, e_1 , and \sqrt{D} .

6. If the biquadratic

$$ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0 \tag{1}$$